Celebrating Torricelli

On the Occasion of His 400th Birthday

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Outline

1. Torricelli's Life
2. Torricelli's Mathematics
   - Torricelli's Trumpet
   - The Cycloid and the Spiral
   - The Isogonic Center
   - Physics and Philosophy
3. References

Brief Biography of Evangelista Torricelli

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- Died, October 25, 1647

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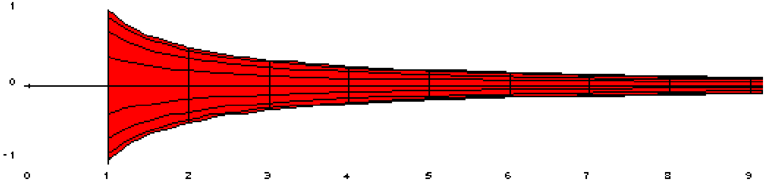
The Trumpet



Finite volume, infinite surface area

The Trumpet

Volume:

$$\pi \int_1^\infty \left(\frac{1}{x}\right)^2 \, dx = \pi \lim_{b\to\infty} \int_1^b \frac{1}{x^2} \, dx = \pi \lim_{b\to\infty} -\frac{1}{x}\Big|_1^b$$

which converges to π.

Torricelli used infinitesimals and the fact that the series $\frac{1}{x^2}$ converges.

The Trumpet

Surface Area:

$$2\pi \int_1^\infty \frac{1}{x}\sqrt{1+\left(-\frac{1}{x^2}\right)^2}\,dx = 2\pi \lim_{b\to\infty}\int_1^b \frac{\sqrt{1+x^4}}{x^3}\,dx$$
$$= 2\pi \lim_{b\to\infty} \frac{1}{2}\sinh^{-1}(x^2) - \frac{\sqrt{x^4+1}}{2x^2}\Bigg|_1^b$$

which diverges.

The Trumpet

Torricelli compared with $\frac{1}{x}$. Knowing that the harmonic series diverges, we have, since

$$\frac{1}{x}\sqrt{1+\frac{1}{x^4}} > \frac{1}{x},$$

that the series $\frac{1}{x}\sqrt{1+\frac{1}{x^4}}$ diverges also.

Thus, a solid with volume π and infinite surface area.

The Trumpet

- Discovered 1643, published 1644
- Totally unexpected
- Reconsideration of "infinity" began
- *"To understand this for sense, it is not required that a man should be a geometrician or a logician, but that he should be mad."* —Thomas Hobbs, 1672

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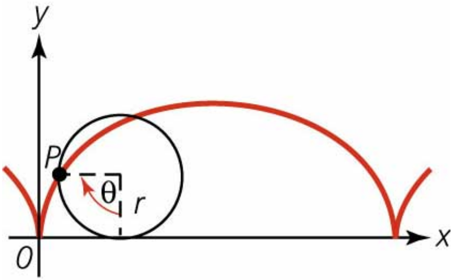
The Cycloid

The *cycloid* is the curve traced by a fixed point P on a circle as the circle rotates along a line.



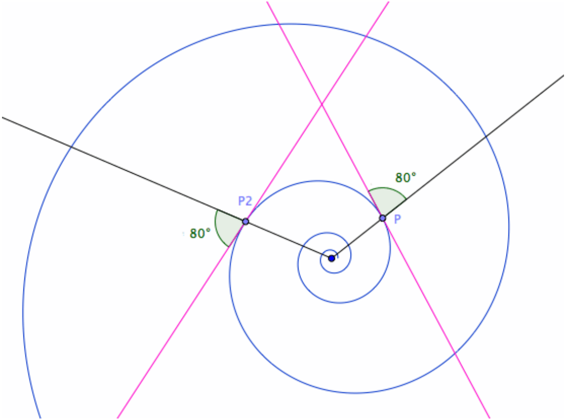
The Cycloid

- Discovery of the curve credited to Torricelli
- Galileo conjectured that area under one arch is nearly 3 times the area of the generating circle
- Torricelli was first to publish proof that $A = 3\pi r^2$ in 1644
- Became the most studied curve of the 17th century

The Equiangular Spiral

Curve characterized by the property that at each point P on it the tangent at P and the radius intersect at the same angle.
Modern polar equation is $r = ae^{b\theta}$

The Equiangular Spiral

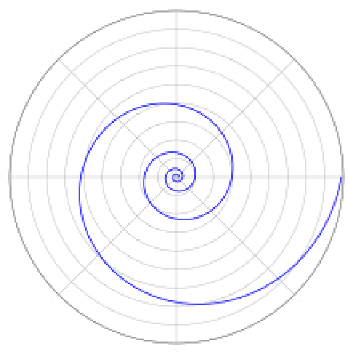


The Equiangular Spiral

The problem: determine its length

- Mathematicians split; some believed it could not be done
- *"... ratios between straight lines and curved lines are not known, and I believe cannot be discovered by human minds."* — Rene Descartes, 1637
- Galileo believed it could be done

The Equiangular Spiral



- Torricelli adapted infinitesimal idea
- Length of spiral from P on x-axis to origin equals tangent from P to y-axis
- First curve (other than the circle) whose length was determined, 1645

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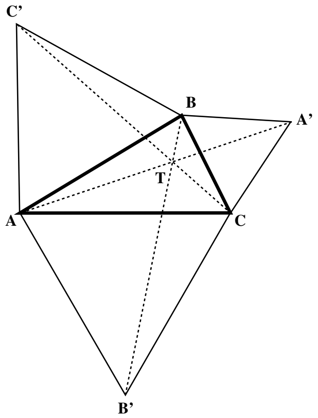
The Torricelli Point

Problem propsed by Fermat:
Determine the point in the plane of a triangle so that the sum of its distances from the vertices is a minimum.

The Torricelli Point

- Solved by Fermat and Torricelli
- Construct equilateral triangles on the three sides and connect corresponding vertices; intersection is the solution
- If $\triangle ABC$ has $\angle A \geq 120°$, then A is the solution.



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The Barometer

- Aristotle: a vacuum is a logical contradiction
- Renaissance scientists altered this to "nature abhors a vacuum"
- Galileo conjectured that a vacuum creates a force

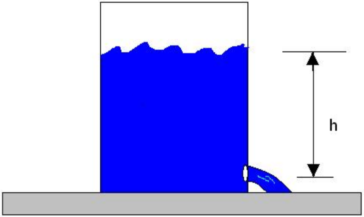
The Barometer

"We have made many glass vessels … with tubes two cubits long. These were filled with mercury, the open end was closed with the finger, and the tubes were then inverted in a vessel where there was mercury … We saw that an empty space was formed and that nothing happened in the vessel where this space was formed … I claim that the force which keeps the mercury from falling is external and that the force comes from outside the tube." — Torricelli, 1644

The Barometer

- Torricelli was the first to accurately explain a vacuum
- He experimented with motion in a vacuum
- $1/760$ of an atmosphere $= 1$ torr

Torricelli's Law



In 1644, experiments with fluids led to Torricelli formulating that the velocity of water is

$$v = k\sqrt{2gh}.$$

Torricelli's Equation

Torricelli formulated the equation of the velocity of an object with constant acceleration without having a known time interval:

$$v_f^2 = v_i^2 + 2a\Delta d.$$

Torricelli's Equation

We begin with the equations of velocity and displacement:

$$v_f = v_i + at, \qquad d = d_i + v_i t + a\frac{t^2}{2}.$$

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$$v_f = v_i + at, \qquad d = d_i + v_i t + a\frac{t^2}{2}.$$

Squaring both sides of the velocity equations gives

$$v_f^2 = v_i^2 + 2av_i t + a^2 t^2$$

and isolating t^2 from the second equation gives

$$t^2 = 2\frac{d - d_i - v_i t}{a} = 2\frac{\Delta d - v_i t}{a}.$$

Torricelli's Equation

Combining these equations gives

$$\begin{aligned} v_f^2 &= v_i^2 + 2av_it + a^2\left(2\frac{\Delta d - v_it}{a}\right) \\ &= v_i^2 + 2av_it + 2a(\Delta d - v_it) \\ &= v_i^2 + 2a\Delta d \end{aligned}$$

References

"To us his incredible genius seems almost miraculous."

—Marin Mersenne

- The MacTutor History of Mathematics Archive
- Calculus Gems by George Simmons
- An Introduction to the History of Mathematics by Howard Eves