History of Mathematics

A Course for High Schools
(Session #132)

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Outline

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Curriculum

State Course number: 27.05200

Four Major Strands

1. Numbers and Operations
2. Algebra
3. Geometry
4. History

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There is a Data Analysis and Probability strand with only one standard:

MHMD1. Students will compute the ratio of winnings in an interrupted game.

Numbers and Operations

MHMN1. Students will explore and use historical computational methods.

MHMN2. Students will explore the implications of infinite sets of real numbers.

Algebra

MHMA1. Students will explore and use historical methods for expressing and solving equations.

MHMA2. Students will explore abstract algebra and group-theoretic concepts.

MHMA3. Students will use and apply number theoretic concepts.

MHMA4. Students will use the algebraic techniques of Fermat, Barrow, and Newton to determine tangents to quadratic curves.

Geometry

MHMG1. Students will prove geometry theorems.

MHMG2. Students will compute lengths, areas, and volumes according to historical formulas.

MHMG3. Students will explore and prove statements in non-Euclidean geometry.

History

MHMH1. Students will identify Hindu-Arabic numerals as a prime scientific advancement.

MHMH2. Students will describe factors involved in the rise and fall of ancient Greek society.

MHMH3. Students will trace the centers of development of mathematical ideas from the 5th century to the 18th century.

MHMH4. Students will identify the 19th and 20th centuries as the time when mathematics became more specialized and more rigorous.

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Approach to the Course

There are many ways to approach a history of math course!

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My approach is to answer the question

What and who influenced the way we do math today?

Approach to the Course

Based on Howard Eves' books, published by MAA, 1984

Great Moments in Mathematics Before 1650

Great Moments in Mathematics After 1650

Specifics About the Course

Length One-semester

Prerequisite Pre-Calculus, or Calculus taken concurrently

Texts Possibilities:

- Katz, *A History of Mathematics: an Introduction*, 3rd edition, Addison-Wesley
- Eves, *An Introduction to the History of Mathematics*, 6th edition, Brooks/Cole
- Berlinghoff and Gouvêa, *Math Through the Ages*, 3rd edtition, MAA

My syllabus for the course

PRINCIPLES TO ACTIONS · Professionalism · Assessment · Curriculum · Access & Equity · Teaching & Learning · Tools & Technology · WHAT'S IN YOUR TOOLBOX?

Assessment

Problems	13 weekly problem sets, called Weekenders
Readings	Classroom discussions
Tests	Mid-term and final: open book, open notes, open internet
Paper	Biographical research, student's choice
Project	Student's choice from *Math Through the Ages* textbook
SLO	25-question multiple-choice

Outline

The Ancients

Two weeks

- Origins
- Babylon, Egypt, China
- Computations
- Linear algebraic equations
- Inductive Geometry
- Thales, Pythagoras

The Ancients: Lectures

- *Scratches and Grunts* – origins of counting
- *First Computations* – arithmetic, false position
- *Greek Mystery* – origins of geometry
- *First Great Theorem* – Pythagoras

Weekender 1, Number 8: Sagittas and Chords

(a) Interpret the following, found on a Babylonian tablet dating from 2600 BC:

> *"60 is the circumference, 2 is the sagitta, find the chord."*
> *"Thou, double 2 and get 4, dost thou not see? Take 4 from 20, thou gettest 16. Square 20, thou gettest 400. Square 16, thou gettest 256. Take 256 from 400, thou gettest 144. Find the square root of 144. 12, the square root, is the chord. Such is the procedure."*

(b) Derive a correct formula for the area of a circular segment in terms of the chord c and sagitta s of the segment.

Weekender 2, Number 2: The Seqt of a Pyramid

The Egyptians measured the steepness of a face of a pyramid by the ratio of the “run” to the “rise”—that is, by giving the reciprocal of what we consider the slope of the face of the pyramid. The vertical unit (the “run”) was the *cubit* and the horizontal unit (the “rise”) was the *hand*; there were 7 hands in a cubit. With these units, the measure of steepness was called the *seqt* of a pyramid.

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(a) Solve Problem 56 of the *Rhind Papyrus* which asks: What is the seqt of a pyramid 250 cubits high and with a square base 360 cubits on a side?

Outline

The Greeks

Four weeks

- Discovery of irrationals
- Origin of Axiomatics
- Euclid's *Elements*
- Archimedes, Ptolemy, Diophantus

The Greeks: Lectures

- *First Crisis...* – proportion and irrationals
- *...And Its Resolution* – Eudoxus
- *Order from Chaos* – axiomatics
- *Mathematician's Bible* – Euclid's *Elements*
- *Thinker and Thug* – Archimedes
- *A Look Toward the Heavens* – astronomy
- *First Great Number Theorist* – origins of algebra

Weekender 6, Number 4: Diophantus Got 99 Problems But Algebra Ain't 1

A common technique employed by Diophantus to solve a system of equations is the following. Consider the system

$$\begin{cases} x + y = a \\ x^2 + y^2 = b. \end{cases}$$

Diophantus sets $x = \frac{a}{2} + z$ and $y = \frac{a}{2} - z$. (Note that this ensures that $x + y = a$.) He then substitutes these expressions in the second equation; this results in a single quadratic in the new variable z, whose solution is straightforward. Having found z, it is then easy to produce the values of x and y.

Outline

The Middle Ages

Two weeks

- Evolution of algebra
- Evolution of numerals
- Islam, China, India
- Fibonacci

The Middle Ages: Lectures

- *Better Scratches, Fewer Grunts* – abacus, numerals
- *Poet-Mathematician* – Khayyam’s solution of the cubic
- *State of the World* – history lesson

Weekender 8, Number 4: Ten Into Two Parts

As a small example of how similar problems occur in different cultures, solve the following two "broken bamboo" problems.

(b) "I have divided 10 into two parts; I multiply the one by 10 and the other by itself, and the products were the same. Find the two parts." *(al-Khwarizmî, c.825)*

(d) "Divide 10 into two parts such that is one squares the first, subtracts it from 97, and takes its square root, then squares the second, subtracts it from 100, and takes its square root, the um of the two roots is 17. Find the two parts." *(de'Mazzinghi, c.1380)*

Outline

The Renaissance

Two weeks

- Solution of cubics
- Logarithms
- Astronomy
- Analytic geometry
- Probability

The Renaissance: Lectures

- *First Controversy* – solution of the cubic
- *Napier's Bones* – logarithms
- *Shoulders of Giants* – Copernicus, Kepler, Galileo
- *Fly on the Wall* – probability

Solutions to Cubics

Cardano solves $x^3 + mx = n$ by considering the identity

$$(a-b)^3 + 3ab(a-b) = a^3 - b^3.$$

If we choose a and b so that $3ab = m$ and $a^3 - b^3 = n$, then x is given by $a - b$. But, solving the last two equations for a and b, one has

$$a = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}, \quad b = \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

so that

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

Weekender 9, Number 6: Galileo's Strange Remark

Explain the remark in Galileo's *The Two New Sciences* that

> *"neither is the number of squares less than the totality of all numbers, nor the latter greater than the former."*

Outline

The Moderns

Two weeks

- Calculus
- Euler
- Lagrange, Laplace, Legendre, Fourier
- Gauss

The Moderns: Lectures

- *Fluent in Fluxions* – calculus
- *Master of Us All* – Euler
- *French Revolution* – French mathematicians
- *Prince of Mathematicians* – Gauss

Weekender 10, Number 9: Euler's Deranged Rearrangements

Euler was posed the following problem: "Given any series of n letters a, b, c, d, etc., to find how many ways they can be arranged so that none returns to the position it initially occupied." Euler called this a *derangement*. If $\Pi(n)$ is the number of derangements of n objects, then $\Pi(1) = 0,\ \Pi(2) = 1,\ \Pi(3) = 2,\ \Pi(4) = 9,\ \Pi(5) = 44$. Euler discovered a general formula:

$$\Pi(n) = n!\left(\frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right)$$

Gauss' Law of Quadratic Reciprocity

First, some notation: the *Legendre symbol* $\left(\frac{p}{q}\right)$ is defined to be 1 if p is a quadratic residue mod q, and -1 if not. Legendre proved the straight-forward product rule for the Legendre symbol: $\left(\frac{ab}{q}\right) = \left(\frac{a}{q}\right)\left(\frac{b}{q}\right)$.

Theorem If p and q are distinct odd primes, then $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$.

Gauss' Law of Quadratic Reciprocity

The more useful form of this theorem is $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)(-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$. To determine whether 19 is or is not a quadratic residue mod 61, we compute the Legendre symbols, reducing by mods each time:

$$\left(\frac{19}{61}\right) = \left(\frac{61}{19}\right)(-1)^{30\cdot 9} = \left(\frac{61}{19}\right) = \left(\frac{4}{19}\right) = \left(\frac{2}{19}\right)\left(\frac{2}{19}\right) = 1$$

Weekender 11, Number 4: Gaussian Integers

Which of the following real primes are primes as Gaussian integers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37? Demonstrate a factorization of those that are composite Gaussian integers.

Outline

Abstraction

Four weeks

- Non-Euclidean geometry
- Non-commutative algebra
- Groups
- The *Erlanger Programme*
- Formal axiomatics
- Cantor’s set theory
- Metamathematics
- Gödel’s incompleteness theorem

Abstraction: Lectures

Two weeks

- *Second Crisis...* – parallel postulate
- *...And Its Non-Resulotion* – non-Euclidean geometry
- *Flash of Genius* – quaternions
- *Another Infinity* – Cantor
- *Mathematics As Theology* – Gödel and foundations
- *New From Old* – hyperreals, four-color theorem, Riemann hypothesis, communication theory
- *Mathematician's Apology* – left-overs

Weekender 12, Number 5: Snarks and Trones

Consider the following set of postulates about certain objects called *trones*, and certain collections of trones called *snarks*.

1. Every snark is a collection of trones.
2. There exist at least two trones.
3. If p and q are two trones, then there exists exactly one snark containing both p and q.
4. If M is a snark, then there exists a trone not in M.
5. If M is a snark, and p is a trone not in M, then there exists exactly one snark containing p and not containing any trone that is in M.

Weekender 12, Number 5: Snarks and Trones

(a) Interpret snark as "committee" and trone as "committee member." Are the postulates consistent?

(b) Interpret trone as any one of the three letters *a*, *b*, *c*, and snark as any one of the three pairs *ab*, *ac*, *bc*. Are the postulates consistent?

(c) Consider the postulates under the following interpretation: a trone is a point and a snark is a line.

Weekender 13, Number 8: Intuition Vs. Abstraction

One purpose of formal axiomatics, with its abstraction and symbolism, is to furnish a barrier against the use of intuition. Answer the following questions intuitively, and then check your answers by calculation.

(b) A man sells half his apples at 3 for 17 cents and then sells the other half at 5 for 17 cents. At what rate should he sell all his apples in order to make the same income?

(d) A clock strikes six in 5 seconds. How long will it take to strike twelve?

Weekender 13, Number 8: Intuition Vs. Abstraction

(e) A bottle and a cork together cost $1.10. If the bottle costs a dollar more than the cork, how much does the cork cost?

(f) Two jobs have a starting salary of $60,000 per year with salaries paid every six months. One job offers an annual raise of $8000 and the other job offers a semiannual raise of $2000. Which is the better paying job?

(h) Four-fourths exceeds three-fourths by what fractional part?

Questions?

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This was session #132